AD-A031 169 NAVAL RESEARCH LAB WASHINGTON D C F/G 20/8

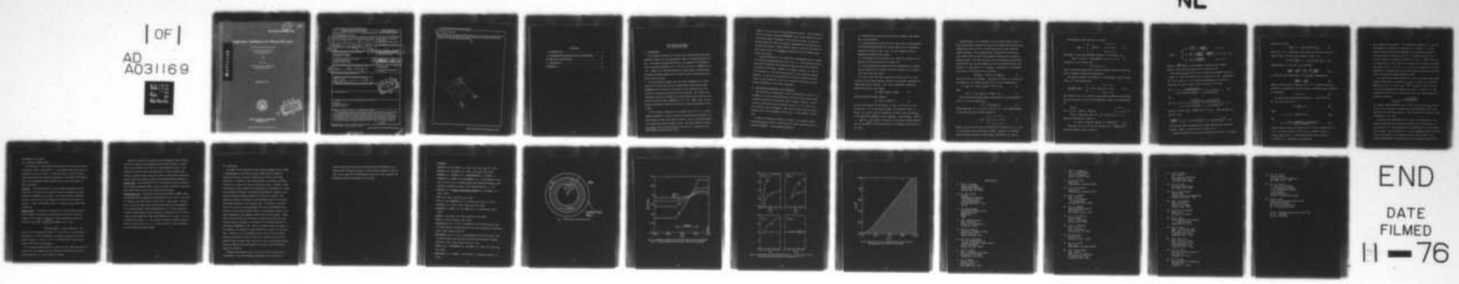
RIGID ROTOR EQUILIBRIUM OF A STRONG ION LAYER.(U)

SEP 76 C A KAPETANAKOS, J GOLDEN, D R CHU

UNCLASSIFIED NRL-MR-3370 NL

1 OF 1

AD
A031169



END

DATE
FILMED
11 — 76

12 FG.

NRL Memorandum Report 3370

Rigid Rotor Equilibrium of a Strong Ion Layer

C. A. KAPETANAKOS AND J. GOLDEN

Experimental Plasma Physics
Plasma Physics Divison

and

K. R. CHU

Science Applications, Incorporated
McLean, Virginia 22101

September 1976

NAVAL RESEARCH LABORATORY
Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
NRL Memorandum Report 3370

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
(6) RIGID ROTOR EQUILIBRIUM OF A STRONG ION LAYER.

5. TYPE OF REPORT & PERIOD COVERED
Interim report on a continuing NRL problem.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
(10) C.A. Kapetanakos, J. Golden [redacted] D.R. Chu

8. CONTRACT OR GRANT NUMBER(s)
(9) Memorandum rept.

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Research Laboratory
Washington, D.C. 20375

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
NRL Problem H02-28A

11. CONTROLLING OFFICE NAME AND ADDRESS
Office of Naval Research
Arlington, Virginia 22217

12. REPORT DATE
(11) Sep[redacted]76

13. NUMBER OF PAGES
24

(12) 20p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
(14) NRL-MR-3370

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.
(16) NRL-H02-28A

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Ion layers
Equilibrium of ion layers
Rotating ion beams

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The equilibrium properties of a strong, space charge neutral ion layer that is confined radially by an externally applied magnetic field are examined within the framework of the steady-state Vlasov-Maxwell equations. The selected distribution function not only allows radial confinement of the gyrating protons, but also is simple enough that explicit, analytic expressions can be obtained for the particle density, current density, magnetic field and the radii of the p-layer, without any a priori assumption about the radial dimensions of the layer. When the ion density is high, thin

(Continues)

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251950

20. Abstract (Continued)

layer equilibria exist only for small rotational velocities, and for a small range of the conducting wall radius, the frequency of rotation can be considerably greater than the cyclotron frequency corresponding to the externally applied magnetic field.

CONTENTS

RIGID ROTOR EQUILIBRIUM
OF A STRONG ION LAYER

I. INTRODUCTION

The conceptual simplicity of rigid rotor equilibria make them very attractive models for the theoretical studies of charged particles layers. Such rigid rotor equilibria within the framework of the Maxwell-Vlasov equations (DAVIDSON et al., 1969, 1970 and 1974 ; ANDERSON et al., 1971 ; MARX, 1968) have been applied in recent years to magnetically confined nonneutral plasmas and to electron layers associated with the Astron reactor concept.

In the nonneutral plasmas case, the model (DAVIDSON et al., 1969, 1970 and 1974) was based on the nonrelativistic limit and the axial diamagnetic field as well as the effect of the conducting wall surrounding the gyrating electrons has been neglected. By contrast, the diamagnetic field was included in the theoretical model studied in connection with the E-layer (ANDERSON et al., 1971 ; MARX, 1968), but the assumed distribution function did not allow radial confinement of the E-layer.

In this paper, we make use of the steady state ($\partial/\partial t = o$), Vlasov-Maxwell equations to study the rigid rotor equilibrium properties of a space charge neutral proton layer (p-layer), that is confined radially by an externally applied magnetic field. The main difference between the present work and that previously carried out in connection to E-

Note: Manuscript submitted September 3, 1976.

layers is in the choice of the distribution function. The selected distribution function (CHU and KAPETANAKOS, 1975) in the present work not only allows radial confinement of the gyrating protons, but also is simple enough that explicit, analytic expressions can be obtained for the particle density, current density, magnetic field and the radii of the p-layer, without any a priori assumptions about the radial dimensions of the layer.

A brief description of the equilibrium configuration and an outline of the general assumptions are given in Sec. II. The equilibrium properties are calculated for the specific choice of the proton distribution function $f_i^o(H-\omega P_\theta) = (m\bar{n}/2\pi)\,\delta(H-\omega P_\theta-k_1)$, where m, $\bar{n}$, ω and k_1 are constants, H is the total energy and P_θ is the canonical angular momentum. In Sec. III, we review briefly results related to the stability of the p-layer and in Sec. IV we discuss the relevance of the present equilibrium to pertinent experiments.

II. EQUILIBRIUM CONFIGURATION AND ASSUMPTIONS

The equilibrium configuration is shown schematically in Fig. 1. It consists of a space charge neutralized, infinitely long, proton layer confined radially by an externally applied magnetic field B_o. We introduce a cylindrical polar coordinate system (r, θ, z), with the z axis along the axis of symmetry. The p-layer rotates about the axis of symmetry with a mean azimuthal velocity $V_\theta^o(r)$ in the negative θ-direction.

To make the theoretical analysis tractable, the following simplifying assumption are made in describing the p-layer equilibrium by the steady-state $\left(\frac{\partial}{\partial t}\right) = o$ Vlasov-Maxwell equations:

(a) Equilibrium properties are azimuthally symmetric and independent of z-coordinate.

(b) The equilibrium electric field of the p-layer is neutralized by an electron background and the current carried by the background electrons is equal to zero. Thus, the self magnetic field is generated entirely by the p-layer.

(c) The magnetic flux enclosed by the conducting cylinder is conserved for the entire life of the layer, and

(d) The equilibrium distribution function describing the p-layer does not depend upon the axial velocity of protons, i.e., there is not motion along the z-axis.

It is well known, that any distribution function that is a function only of the single particle constants of the motion satisfies the steady-state Vlasov equation. For a time independent, azimuthally symmetric field the total energy

$$H = mv_r^2/2 + mv_\theta^2/2 + mv_z^2/2 \,, \tag{1}$$

and the canonical angular momentum

$$P_\theta = mrv_\theta + qrA_\theta^o(r)/c \,, \tag{2}$$

are the two constants of the motion. In eqs. (1) and (2) m is the proton mass, q = e is the proton charge, c the speed of light in vacuum, v_r, v_θ and v_z are the components of proton velocity and $A_\theta^o(r)$ is the equilibrium magnetic vector potential. Note that $A_\theta^o(r) = A_\theta^{ext}(r) + A_\theta^{self}(r)$, where $A_\theta^{ext}(r)$ describes the externally applied magnetic field B_o and $A_\theta^{self}(r)$ describes the axial self magnetic field $B_z^s(r)$ of the layer.

Experimentally, it is desirable to form p-layers without shear in their mean azimuthal velocity of rotation and without all the particles composing the layer having the same energy and the same canonical angular momentum. Such equilibria are in general more stable than those having shear in their mean azimuthal velocity and no energy or momentum spread. For these reasons, we are considering an equilibrium distribution function than depends on H and P_θ through the linear combination, $H - \omega P_\theta$, where ω is a constant. Specifically, the distribution function is assumed to be of the form

$$f_i^o(H-\omega P_\theta) = (m\bar{n}/2\pi)\ \delta\ (H-\omega P_\theta-k_1), \tag{3}$$

where m, $\bar{n}$ and k_1 are constants. Using Eq. (1) with $v_z = o$ and Eq. (2), the argument of the delta function in Eq. (3) may be expressed as

$$H - \omega P_\theta - k_1 = (m/2)\ [(v_\theta-r\omega)^2 + v_r^2] + U(r), \tag{4}$$

where

$$U(r) = -\ (qr/c)\ A_\theta^o(r)\omega - mr^2\omega^2/2 - k_1\ . \tag{5}$$

The equilibrium density profile corresponding to the distribution of Eq. (3) can be computed from

$$n^o(r) = \int f_i^o(H-\omega P_\theta)d^2v. \tag{6}$$

Substituting Eqs. (3), (4) and (5) into Eq. (6), it is straightforward to show that the proton density profile is

$$n^o(r) = \begin{cases} o,\ o \le r < a_1\ , \\ \bar{n},\ a_1 \le r \le a_2, \\ o,\ a_2 < r < b\ , \end{cases} \tag{7}$$

where a_1 and a_2 are the roots of function U(r) defined in Eq. (5), and b is the radius of the conducting cylinder. Equation (7) indicates that the density profile has sharp radial boundaries at a_1 and a_2.

The azimuthal current density is given by

$$J_\theta^o(r) = \begin{cases} o, & o \le r < a_1 \ , \\ q\bar{n}\ V^o(r) & a_1 \le r \le a_2, \\ o & a_2 < r \le b \ , \end{cases} \tag{8}$$

where $V_\theta^o(r)$ is the mean azimuthal velocity of the p-layer, i.e.,

$$V_\theta^o(r) = \langle v_\theta(r)\rangle = (1/\bar{n})\int v_\theta f_i^o d^2v = \omega r, \ a_1 \le r \le a_2. \tag{9}$$

Similarly, it can be shown that

$$V_r^o = \langle v_r(r)\rangle = o.$$

Thus, the p-layer rotates with a constant angular velocity ω around its axis of symmetry (rigid-rotor equilibrium).

The vector potential corresponding to the azimuthal current density of Eq. (8) is determined from

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\left[rA_\theta^o(r)\right] \begin{cases} o & , o \le r < a_1 \ , \\ -(4\pi/c)\ q\bar{n}r\omega, & a_1 \le r \le a_2, \\ o & , a_2 < r < b \ . \end{cases} \tag{10}$$

The solution of Eq. (10) subject to the boundary conditions that A_θ^o and B_z are continuous at a_1 and a_2 and $\int_o^b 2\pi B_z r dr = \pi b^2 B_o$ (flux conservation) is

$$A^o(r) = \begin{cases} k_2 r, & o \le r \le a_1 \\ k_2 r - \pi\bar{n}q\omega(r^2 - a_1^2)^2/2cr, & a_1 \le r \le a_2, \\ k_2 r - \pi\bar{n}q\omega\ \left[(a_2^2 - a_1^2)\ (2r^2 - a_1^2 - a_2^2)\right]/2cr, & a_2 < r \le b, \end{cases} \tag{11}$$

where the constant k_2 is given by

$$k_2 = B_o/2 + (\pi\bar{n}q\omega/2c)\ (a_2^2 - a_1^2)\ \left[2 - (a_1^2 + a_2^2)/b^2\right] \ . \tag{12}$$

Using Eqs. (11) and (12) and the relation $B^o(r) = \frac{1}{r}\frac{\partial}{\partial r}\left[rA_\theta(r)\right]$, the axial magnetic field is given by

$$B_z^o(r) = \begin{cases} B_o + \dfrac{4\pi I_\ell}{c}\left[1 - \dfrac{(a_1^2+a_2^2)}{2b^2}\right] & , o \le r < a_1 \\ B_o + \dfrac{4\pi}{c} I_\ell \left[\dfrac{(a_2^2-r^2)}{(a_2^2-a_1^2)} - \dfrac{(a_1^2+a_2^2)}{2b^2}\right], & a_1 \le r \le a_2, \\ B_o - \dfrac{4\pi}{c} I_\ell \left[\dfrac{(a_1^2+a_2^2)}{2b^2}\right] & , a_2 < r \le b , \end{cases} \quad (13)$$

where $I_\ell = \frac{q\bar{n}\omega}{2}(a_2^2-a_1^2)$ is the azimuthal current per unit length. Figure 2 shows the ratio $B_z^o(r)/B_o$ as a function of r/b.

The inner and outer radii of the p-layer are determined from $U(a_1) = U(a_2) = o$, where the function $U(r)$ is given by Eq. (5). Substituting Eq. (11) into Eq. (5), it is rather straightforward but tedious to show that the radii a_1 and a_2 can be expressed in the form

$$a_1^2 = - \frac{16k_1\lambda^{*2}/(m\omega^2b^2)}{1 \pm \left\{1 + 16(\lambda^*/b)^2\left[(\frac{\Omega_o}{\omega}+1) + 4k_1\Big/(m\omega^2b^2)\right]\right\}^{1/2}} , \quad (14)$$

and

$$a_2^2 = b^2 + \frac{8\lambda^{*2}\left[(\frac{\Omega_o}{\omega}+1) + 2k_1\Big/(m\omega^2b^2)\right]}{1 \pm \left\{1 + 16(\lambda^*/b)^2\left[(\frac{\Omega_o}{\omega}+1) + 4k_1\Big/(m\omega^2b^2)\right]\right\}^{1/2}} , \quad (15)$$

where $\Omega_o = qB_o/mc$, $\omega_{pi}^2 = 4\pi\bar{n}q^2/m$ and $\lambda^* = c/\omega_{pi}$. Note from Eqs. (14) and (15) that

$$\frac{2(a_2^2-a_1^2)}{b^2} = 1 \pm \left\{ 1 + 16(\lambda^*/b)^2\left[(\frac{\Omega_o}{\omega}+1) + 4k_1\Big/(m\omega^2b^2)\right]\right\}^{1/2}, \quad (16)$$

and since $a_2^2 > a_1^2$, the denominator in these equations must always be positive. Then, in order for a_1 to be real, $k_1 \le o$.

Radial confinement of the p-layer requires that $a_2^2 \le b^2$. It follows then from (15) that

$$\left[\left(\frac{\Omega_o}{\omega}+1\right)-2|k_1|\Big/(m\omega^2b^2)\right]\le o, \tag{17}$$

where $|k_1| = -k_1$. Furthermore, since $(a_2^2 - a_1^2)$ should be real, the radical in Eq. (16) should be positive or zero, i.e.,

$$16(\lambda^*/b)^2\left[\left(\frac{\Omega_o}{\omega}+1\right)-4|k_1|\Big/(m\omega^2b^2)\right]\ge -1. \tag{18}$$

Combining Eqs. (17) and (18), it is obtained

$$\frac{4|k_1|}{m\omega^2b^2}-\frac{b^2}{16\lambda^{*2}}\le\left(\frac{\Omega_o}{\omega}+1\right)\le\frac{2|k_1|}{m\omega^2b^2}\quad . \tag{19}$$

In order for both inequalities to be satisfied simultaneously,

$$\frac{2|k_1|}{m\omega^2b^2}\le\frac{b^2}{16\lambda^{*2}}\quad . \tag{20}$$

When the conducting wall is absent, i.e., $b \to \infty$, equilibrium exists, provided ω is negative for positive Ω_o and its absolute value satisfied the inequality

$$o < |\omega| \le \Omega_o\ . \tag{21}$$

Introducing the dimensionless variable $\lambda_o = (2|k_1|/m)^{1/2}\,\Omega_o^{-1}$, Eq. (19) can be written as

$$\omega_1 \le \left(\frac{\omega}{\Omega_o}\right) \le \omega_3\ , \tag{22}$$

where

$$\omega_1 = [-1-(1+4\lambda_o^2/b^2)^{1/2}]/2\ , \tag{23a}$$

and

$$\omega_3 = \frac{-1-\{1+(8\lambda_o^2/b^2)[1+(b/4\lambda^*)^2]\}^{1/2}}{2[1+(b/4\lambda^*)^2]}\quad . \tag{23b}$$

Figure 3 shows ω_1 and ω_3 as a function of b/λ_o for several values of λ^*/λ_o. For small values of λ^*/λ_o (high ion density) there is a small range of b/λ_o, in which equilibrium exists for values of $|\omega|/\Omega_o \gg 1$. In general such equilibria are of appreciable thickness. For intermediate values of λ^*/λ_o (Fig 3c), the frequency of rotation is restricted to $o < |\omega|/\Omega_o < 1$. In the special case that the various parameters of the system are tuned in such a way that the equality of Eq. (20) is satisfied, then $\omega_1 \simeq \omega_3$ and the frequency of rotation is given by Eq. (23a). Finally (Fig. 3d), for large values of λ^*/λ_o (small ion density), the normalized frequency of rotation $(\omega/\Omega_o) \simeq \omega_1 \approx \omega_3 \approx -1$, i.e., as it is expected all the ions rotate with a frequency that is near the cyclotron frequence corresponding to the externally applied magnetic field.

It is easy to be shown from Eq. (16), that in the thin layer approximation the frequency of rotation (ω_{thin}/Ω_o) is given by an expression similar to that of Eq. (23b), with $(\lambda^*)^2$ replaced by $(\lambda^*)^2/\epsilon$, where ϵ is a positive quantity much smaller than unity. If $(b/4\lambda^*) < 1$, the inequalities of Eq. (22) are satisfied and the thin layer frequency of rotation is given by

$$\left(\omega_{thin}/\Omega_o\right) \simeq \frac{-1-\sqrt{1 + 8\lambda_o^2/b^2}}{2} .$$

By contrast, when $\epsilon(b/4\lambda^*)^2 >> 1$ and $\lambda_o/\lambda^* >> 1$, both ω_3 and (ω_{thin}/Ω_o) go to zero, i.e. for high ion density, thin layer equilibria exist only with very low rotational frequency. This conclusion is similar to that of LOVELACE et al., (1974).

In order for the inequalities of Eq. (22) to be satisfied $\omega_3 \geq \omega_1$. Using the expressions for ω_1 and ω_3 from Eq. (23), this relation is plotted in Fig. 4. The inequality is satisfied only in the upper half of the figure. For fixed values of λ_o and b, equilibria exist provided that the ion density in the layer exceeds a critical value. This clearly demonstrates the importance of the self magnetic field in the confinement of the layer.

III. REVIEW OF INSTABILITIES

The stability of the equilibrium discussed in the previous section is presently under investigation. It is expected that results will be available in the near future. In the mean time, in order to obtain some insight about the stability properties of the rigid rotor equilibria, we are reviewing several unstable modes that are pertinent to these equilibria.

Diocotron: The diocotron mode in a space charge nonneutral E-layer consisting of electrons that encircle the axis of symmetry has been studied by NOCENTINI et al., (1968). They found that the sufficient condition for stability is the absence of shear in the velocity of rotation. Thus, the diocotron mode is not present in any rigid rotor equilibrium.

Negative mass: The stability condition for a space charge neutral E-layer composed of relativistic electrons that encircle the axis of symmetry and confined in a magnetic field configuration that has a weak field index n $\left[= -\frac{r}{B_z}\frac{\partial B_z}{\partial r}\right]$ is (LANDAU et al., 1966)

$$\left[\Delta/(2m_o c r_o)\right]^2 > \nu g\gamma/\alpha, \text{ (stability)}, \quad (24)$$

where m_o is the electron rest mass, $g = c^2L/(2\pi r_o)$, r_o is the equilibrium radius, L is the inductance of the beam in cgs units, Δ is the momentum spread $\alpha = 1/(1-n) - \gamma^{-2}$, $\nu = Ne^2/(2\pi r_o m_o c^2)$, and N is the number of electrons in the system.

For a strong, nonrelativistic p-layer, both the field index and the coefficient α are negative and the stability condition [Eq. (24)] is always satisfied, i.e., the p-layer is stable.

However, it should be noticed that the theoretical work of Landau and Neil is based on the assumption that the field index n is small, which is not probably the case for a field reversing p-layer. Thus definite conclusions about the stability of a strong p-layer with respect to negative mass instability cannot be drawn before recalculating the stability condition without the weak field index assumption.

Tearing mode: This instability is not of any great concern, because it can be easily stabilized (MARX, 1968) by placing a perfectly conducting wall sufficiently close to the charged particle layer.

Precessional mode: It has been predicted theoretically (FURTH, 1965) and demonstrated experimentally (CHRISTOFILOS et al., 1969) in the case of E-layers, that this mode is present only in weak layers. Furthermore, it has been shown that the precessional mode can be stabilized by a toroidal magnetic field (CHRISTOFILOS et al., 1972) produced by currents flowing along an axial conductor or cantilever or by a sufficiently strong quadrupole Ioffe field (WOODALL et al., 1975). Thus, it is expected this mode is only present whenever the layer strength is below a critical level, i.e., it is only of small, if any, importance for field reversing proton layers.

IV. CONCLUSIONS

The number of ions required to form a field reversing ring or layer is approximately by the factor (m_i/m_e) higher than the number of electrons required to form an electron ring or layer of the same dimensions. In addition, the current density of presently available ion sources is lower than those of electron sources. Therefore, field reversing ion rings or layers can be realized by using hollow ion beams of considerable dimensions and passing them through a magnetic cusp. Since the canonical angular momentum is a function of radius, such methods of injection correspond to an appreciable spread in the canonical angular momentum of the injected ions. Furthermore, the force that an ion sees as it passes through the cusp is approximately proportional to its radial distance and thus an appreciable spread in the energy transverse to the magnetic field lines will be present. Therefore, only equilibria that are based on distributions functions that include spread in P_θ and $E_\perp$ are pertinent to the present methods of injections (KAPETANAKOS et al., 1976) of intense, pulsed ion beams. In this respect, our selection of the distribution function is very realistic. However, it is not presently clear which must be the exact injection conditions in order to form an ion layer in which all the particles have the same $H-\omega P_\theta$. But even if such conditions were known, transient effects during the buildup of the layer could drastically change the situation.

Although the assumption that all the ions have the same $H-\omega P_\theta$ is questionable, it has the important advantage that it results to a simple distribution function that allows explicit expressions to be obtained for the particle density, current density, magnetic field and radii of the layer. Such a distribution function is very convenient for studying the stability properties of the layer.

REFERENCES

DAVIDSON, R.C. and KRALL, N.A., Phys. Rev. Letts. 23, 833 (1969).

DAVIDSON, R.C. and KRALL, N.A., Phys. Fluids 13, 1543 (1970).

ANDERSON, D.V., BENFORD, G.A., BRETTSCHNEIDER, M., KILLEEN, J., NEWCOMB, W.A., MIRIN, A.A. and RENSINK, M.E., in Proceedings of the Fourth Inter. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, 1971 (IAEA-CN-2B/A-10.) p. 137.

DAVIDSON, R.C., Theory of Nonneutral Plasmas (W.A. Benjamin, Inc., Mass., 1974).

MARX, K.D., Phys. Fluids 11, 357 (1968).

CHU, K.R. and KAPETANAKOS, C.A., Nuclear Fusion 15, 947 (1975).

LOVELACE, R.V., et al., Phys. Fluids 17, 2081 (1974).

NOCENTINI, A., BERK, H.L. and SUDAN, R.N., J. Plasma Phys. 2, 311 (1968).

LANDAU, R. and NEIL, V.K., Phys. Fluids 9, 2412 (1966).

FURTH, H., Phys. Fluids 8, 2020 (1965).

CHRISTOFILOS, N.C., et al., in Proceedings of the Third Inter. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, 1968, (IAEA, Vienna, 1969) 967.

CHRISTOFILOS, N.C., et al., in Proceedings of the Fourth Inter. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, 1971. (IAEA, Vienna, 1972), p. 119.

WOODALL, D.M., FLEISCHMANN, H.H. and BERK, H.L., Phys. Rev. Lett. 34, 260 (1975).

KAPETANAKOS, C.A., GOLDEN, J. and YOUNG, F., Nuclear Fusion 16, 151 (1976).

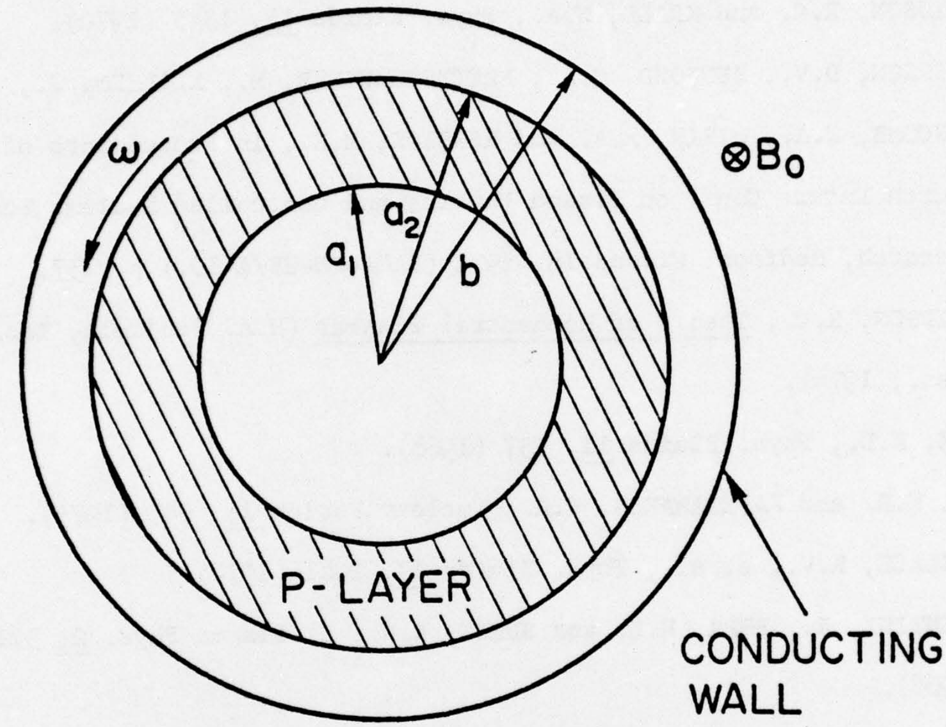


Fig. 1 — Cross section of the long ion layer

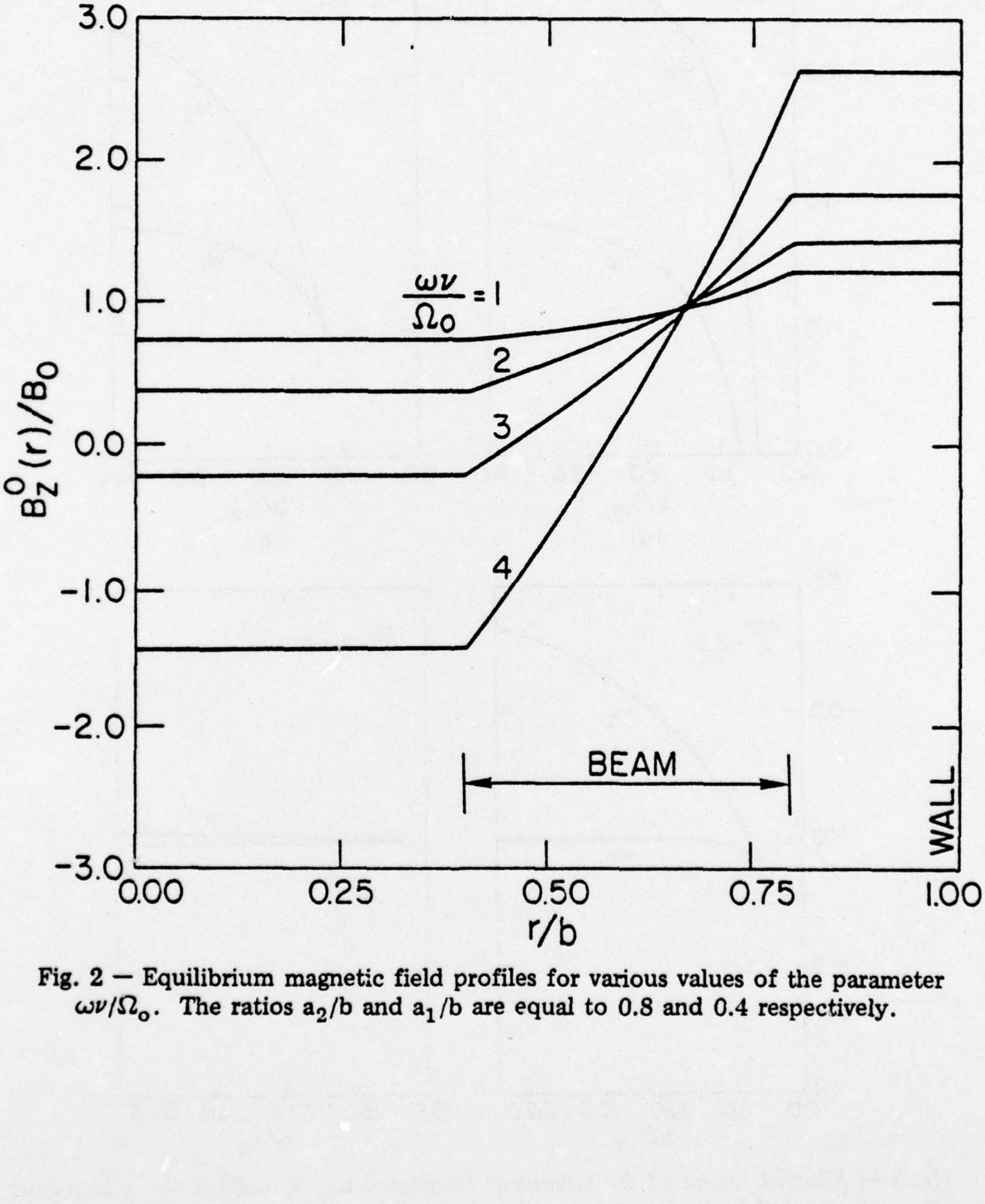


Fig. 2 — Equilibrium magnetic field profiles for various values of the parameter $\omega\nu/\Omega_o$. The ratios a_2/b and a_1/b are equal to 0.8 and 0.4 respectively.

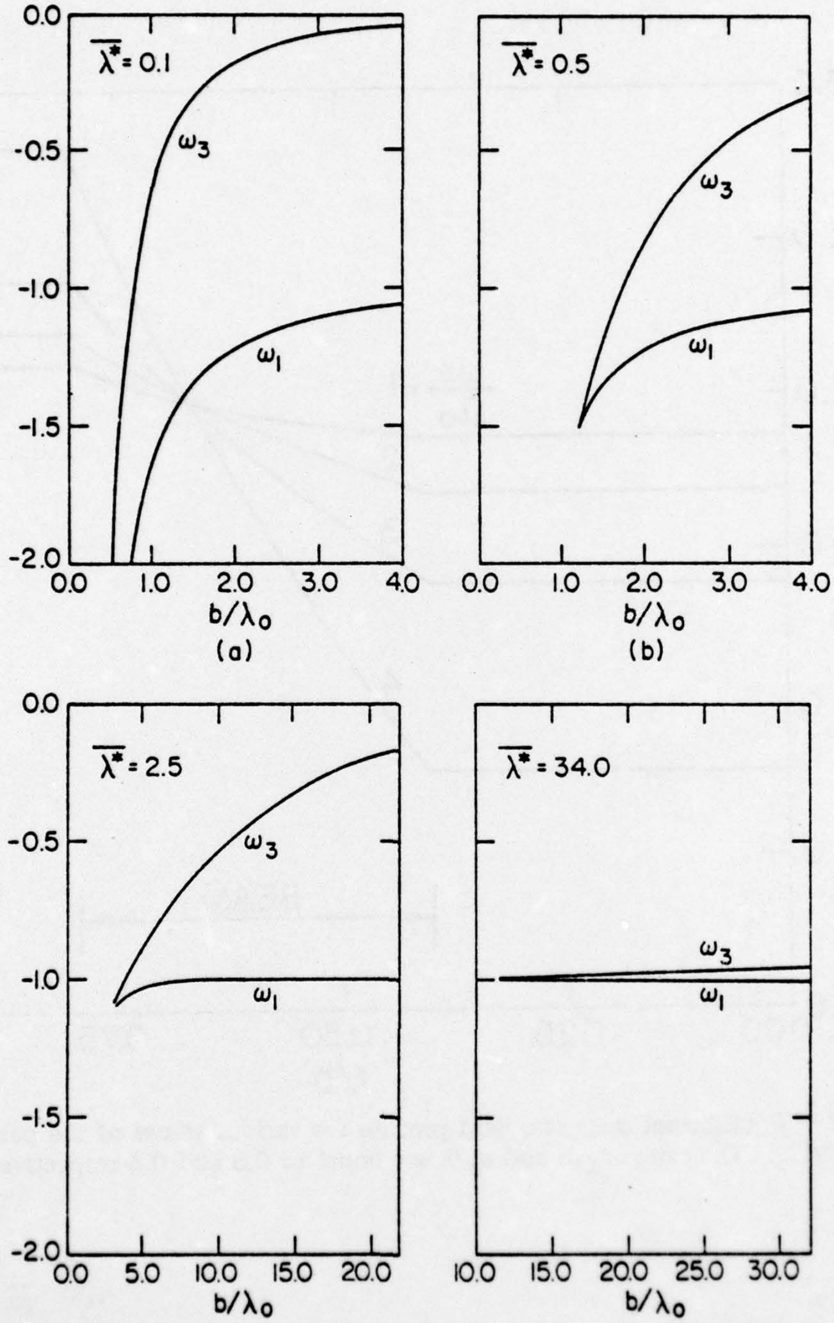


Fig. 3 — Allowed region of the rotational frequency $\omega_1 \leqslant (\omega/\Omega_o) \leqslant \omega_3$ as a function of λ_o/b, for various values of the parameter $\lambda^*/\lambda_o = \overline{\lambda^*}$

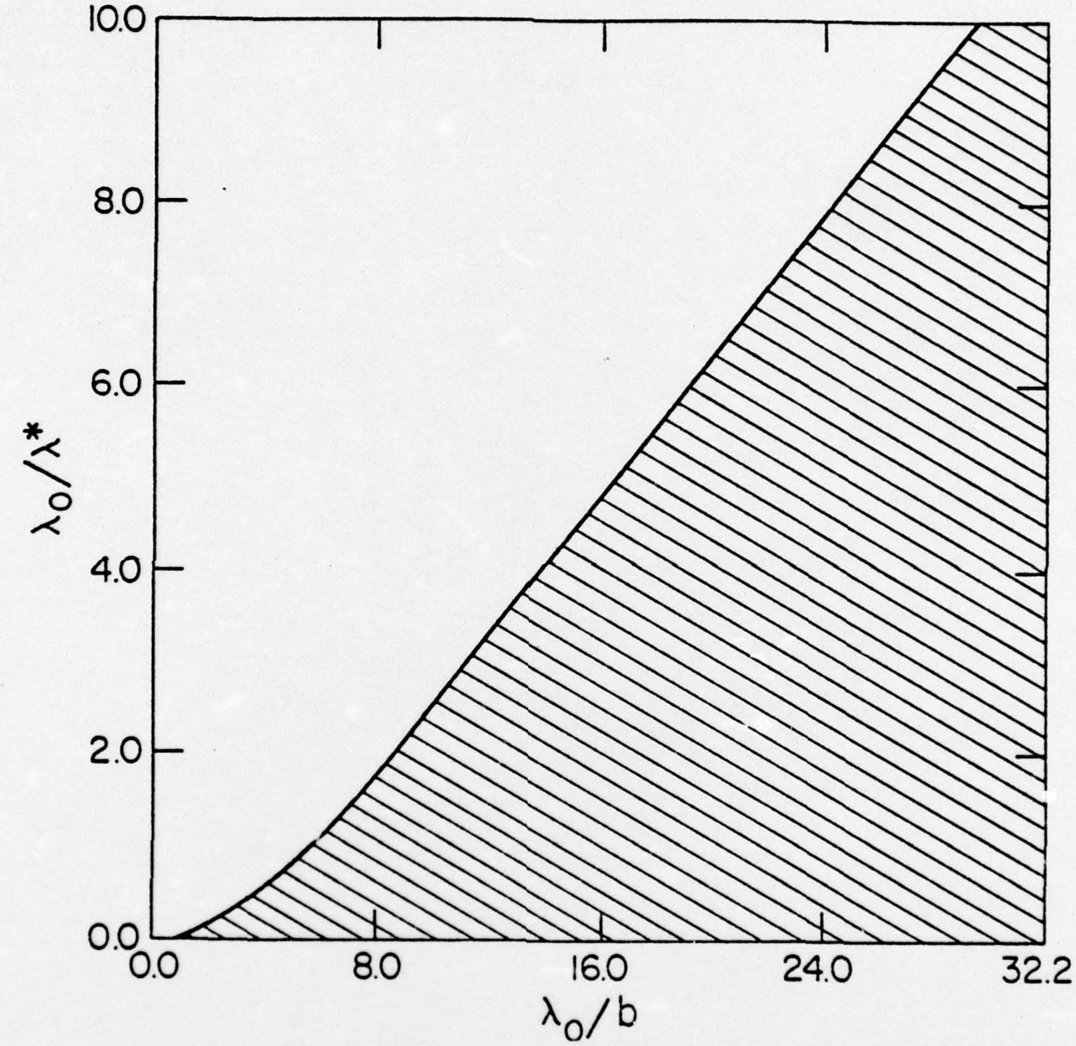


Fig. 4 — Plot of λ_o/λ^* vs. λ_o/b. Equilibria exist only for values of these parameters at the upper half of the figure.

Distribution List

1. Prof. R. Davidson
Physics Department
University of Maryland
College Park, MD 20742

2. Dr. H.J. Doucet
Director
Laboratoire de Physique
des Milieux Ionises
Ecole Polytechnique
17, Rue Descartes, P RIS-V
France

3. Dr. Takaya Kawabe
Institute of Plasma Physics
Nagoya University
Nagoya 464
Japan

4. Prof. Norman Rostoker
Dept. of Physics
University of California
Irvine, CA 92664

5. Prof. C.B. Wharton
Laboratory of Plasma Studies
Cornell University
Ithaca, New York 14850

6. Prof. H.H. Fleichmann
Lab. for Plasma Studies &
School of Applied & Engr. Physics
Cornell University
Ithaca, New York 14850

7. Prof. M. Reiser
Dept. of Physics & Astronomy
University of Maryland
College Park, MD 20742

8. Dr. S. Putnam
Physics Intern. Co.
2700 Merced St.
San Leandro, CA 94577

9. Prof. S. Humphries
Lab. of Plasma Studies
Cornell University
Ithaca, New York 14850

10. Dr. Gerold Yonas
Sandia Lab.
Albuquerque, New Mexico 87115

11. Dr. D. A. McArthur
Sandia Lab.
Albuquerque, New Mexico 87115

12. Prof. N. Luhmann
Dept. of E. E.
Univ. of California
at Los Angeles
Los Angeles, Calif. 90024

13. Dr. Jim Benford
Physics International Co.
2700 Merced St.
San Leandro, Calif. 94577

14. Dr. C. W. Roberson
Physics Dept.
Univ. of California
Irvine, Calif. 92664

15. Prof. C. Striffler
Dept. of E. E.
Univ. of Maryland
College Park, Md. 20742

16. Dr. Peter Korn
Maxwell Labs.
San Diego, Calif. 92123

17. Dr. C. L. Olson
Sandia Lab.
Albuquerque, New Mexico 87115

18. Prof. George Bekefi
Bldg. 36-213
Mass. Inst. of Technology
77 Massachusetts Ave.
Cambridge, Mass. 02139

19. Donald Prosnitz
Bldg. 36-295
Mass. Inst. of Technology
77 Massachusetts Ave.
Cambridge, Mass. 02139

20. Dr. J.A. Rome
Oak Ridge National Lab.
Oak Ridge, Tenn. 37830

21. Prof. George Schmidt
Physics Dept.
Stevens Institute of Technology
Hoboken, New Jersey 07030

22. Prof. W.E. Drummond
Dept. of Physics
University of Texas
Austin, Texas 78712

23. Prof. A.W. Trivelpiece
Maxwell Lab.
San Diego, CA 92123

24. Dr. B. Miller
Energy Research & Development
Administration
Washington, DC 20545

25. Prof. A. Bers
Dept. of Electrical Engr.
Mass. Inst. of Technology
77 Massachusetts Ave.
Cambridge, Mass. 02139

26. Prof. J.E. McCune
Dept. of Aero. & Astro.
Mass. Institute of Tech.
77 Massachusetts Ave.
Cambridge, Mass. 02139

27. Dr. D.S. Prono
Lawrence Livermore Laboratory
P.O. Box 808
Livermore, CA 94550

28. Dr. R. Briggs
Lawrence Livermore Laboratory
P.O. Box 808
Livermore, CA 94550

29. Dr. P.D. Miller
Physics Division
Oak Ridge National Laboratory
Oak Ridge, Tenn. 37830

30. Dr. E.S. Weibel
c/o Center de Recharches
en Physique des Plasmas
Ecole Polytechnique Federale
de Lausanne
Avenue des Bains 21
CH-1007, Lausanne, Switzerland

31. Prof. R. Uzan
Laboratoire D'emission Electronique
Faculte des Sciences
43, Bd du 11 Novembre 1918
69 - Villeurbanne, France

12 cys - Defense Documentation Center (DDC)
20 cys - Code 2628
25 cys - Code 7700